

Capacities and the Choquet integral in decision making: a survey of fundamental concepts and recent advances

Michel Grabisch

Centre d'Economie de la Sorbonne, Université Paris I
106-112, Bd de l'Hôpital, 75013 Paris, France
Email: michel.grabisch.at.univ-paris1.fr

Capacities have been introduced by Choquet in 1953 [1], as well as a functional being known now as the Choquet integral. Independently in 1974, Sugeno introduced the concepts of fuzzy measure [11], which is basically the same as a capacity, and of fuzzy integral, known now as the Sugeno integral. From then, these two very different integrals began to be widely used in applied fields, essentially decision making (more particularly, decision under risk and uncertainty, multicriteria decision aid (MCDA)). If in decision under uncertainty, the use of the Choquet integral immediately received a firm theoretical basis through providing axiomatic characterizations [10], the first works in MCDA remained on a rather intuitive and experimental level. Perhaps the idea of Murofushi of using the Shapley value as an importance index [8], and later his proposal of interaction index [9], were the starting point for a more theoretical basis for the use of Choquet integral in MCDA. This was later developed by the author, proposing the notion of k -additive fuzzy measure [3], and using the MACBETH approach and concepts from measurement theory to embed the whole thing into a firm theoretical basis [5, 6, 7].

This survey will try to explain the fundamental concepts underlying the use of capacities and the Choquet integral in MCDA, as well as some recent advances. It is partly based on the survey papers [2] and the more recent [4], where the interested reader can find many references. We address the following topics:

- (i) measurement scales, unipolar and bipolar scales
- (ii) importance and interaction indices, Shapley value, various indices describing the behavior of the model
- (iii) k -additive capacities, p -symmetric capacities
- (iv) the Choquet integral as a parsimonious linear interpolator.

Lastly, we address the following very recent topic: suppose that the capacity is not known for every subset of the set N of criteria. How to compute the Choquet integral in this case? This situation may arise in MCDA for the following reason. A subset S of N is viewed as a prototypical alternative, being totally satisfactory on criteria in S , and totally unacceptable on other criteria. Depending on the peculiar problem, evidently some of these prototypical alternatives cannot exist, hence the capacity cannot be defined for these subsets. As a consequence, there will exist alternative such that it is not possible to compute their Choquet integral, which means that no overall score can be given to them. We provide an extension of the Choquet integral able to deal with this situation.

References

- [1] G. Choquet. Theory of capacities. *Annales de l'Institut Fourier*, 5:131–295, 1953.
- [2] M. Grabisch. The application of fuzzy integrals in multicriteria decision making. *European J. of Operational Research*, 89:445–456, 1996.
- [3] M. Grabisch. k -order additive discrete fuzzy measures and their representation. *Fuzzy Sets and Systems*, 92:167–189, 1997.
- [4] M. Grabisch and Ch. Labreuche. A decade of application of the Choquet and Sugeno integrals in multi-criteria decision aid. *4OR*, 6:1–44, 2008. doi 10.1007/s10288-007- 0064-2.

- [5] M. Grabisch, Ch. Labreuche, and J.-C. Vansnick. On the extension of pseudo- Boolean functions for the aggregation of interacting bipolar criteria. *Eur. J. of Operational Research*, 148:28–47, 2003.
- [6] Ch. Labreuche and M. Grabisch. The Choquet integral for the aggregation of interval scales in multicriteria decision making. *Fuzzy Sets and Systems*, 137:11–26, 2003.
- [7] Ch. Labreuche and M. Grabisch. Generalized Choquet-like aggregation functions for handling bipolar scales. *Eur. J. of Operational Research*, 172:931–955, 2006.
- [8] T. Murofushi. A technique for reading fuzzy measures (I): the Shapley value with respect to a fuzzy measure. In *2nd Fuzzy Workshop*, pages 39–48, Nagaoka, Japan, October 1992. In Japanese.
- [9] T. Murofushi and S. Soneda. Techniques for reading fuzzy measures (III): interaction index. In *9th Fuzzy System Symposium*, pages 693–696, Sapporo, Japan, May 1993. In Japanese.
- [10] D. Schmeidler. Subjective probability and expected utility without additivity. *Econometrica*, 57(3):571–587, 1989.
- [11] M. Sugeno. Theory of fuzzy integrals and its applications. PhD thesis, Tokyo Institute of Technology, 1974.